Duopoly Competition in Advertising-Sponsored Wi-Fi Provision

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Abstract

It has been increasingly popular for venues (e.g., restaurants) to collaborate with advertisers on the provision of public Wi-Fi services. The venues' visitors can watch the advertisers' ads in exchange for free Wi-Fi access, and the venue owners charge the advertisers for the ad display. In this work, we consider competition in advertising-sponsored Wi-Fi provision. Two venue owners with overlapping coverage compete for users and further sell the ad slots generated by the users to an advertiser. We model the strategic interactions among the venue owners, advertiser, and users as a three-stage game, and analyze the game equilibrium. Our results show that the venue owners' advertising densities affect their market shares as well as the advertiser's overall payment.

1. Introduction

The Wireless Broadband Alliance estimated that 50% of global commercial Wi-Fi hotspots are offered by the venues, e.g., restaurants and stores (see Gabriel 2014). A conventional approach to monetize these Wi-Fi hotspots is to directly charge users for the Wi-Fi usage. However, users prefer to have a free Wi-Fi access, and this monetization approach may simply lead to a low utilization rate of the Wi-Fi hotspots. Recently, many venues have started offering a novel monetization approach, where they request the users to watch ads before using the Wi-Fi service. The venues can sell the ad slots to advertisers to earn revenue. This monetization is effective, since more users can use the Wi-Fi service and the advertisers can also achieve targeted advertising (e.g., deliver ads to the users based on their locations). There have been many online advertising platforms that offer technical support for the venues to offer such advertising-sponsored Wi-Fi services. Examples of these platforms include SOCIFI, Boingo, Purple Wi-Fi, and Wi-Fi Garden.

In this work, we consider the impact of competition between two nearby venue owners offering such advertising-sponsored public Wi-Fi services. Specifically, the venue owners compete to offer the Wi-Fi services to users. Each venue owner decides the advertising density in its Wi-Fi network, i.e., the fraction of the time that a user needs to spend in watching ads when using the network. Moreover, each venue owner chooses the unit ad price. Each user then decides whether to use the Wi-Fi service and which Wi-Fi network to use based on its valuation for the Wi-Fi service, the advertising densities, and the congestion levels of the two networks. Furthermore, an advertiser decides the mass of advertisements to display given the two venue owners' unit ad prices. We model the interactions by a three-stage game, and characterize its equilibrium.

Our analysis in this work leads to the following insights. First, both venue owners' have positive market shares (i.e., positive mass of users using the Wi-Fi service) if and only if their advertising densities are close to each other. When the network capacity increases, the users become more sensitive to the difference in the advertising densities. In this case, even when one venue owner chooses a slightly larger advertising density, it may get a zero market share. Second, the venue owners' unit ad prices are heavily affected by the advertising wear-out effect. If the ad repetition can easily become annoying to the users, a venue owner's optimal unit ad price is independent of the wear-out effect's degree and the advertising density; otherwise, the optimal unit ad price decreases with the wear-out effect's degree and advertising density.

2. Related Work

Yu et al. (2017) studied the comparison between the fee-based and advertising-sponsored Wi-Fi access. There are three major differences between this work and the reference. First, Yu et al. considered a single venue owner's Wi-Fi monetization strategy, and this work focuses on the competition between the venue owners and studies the users' choices between the two venues. Second, Yu et al. assumed that the Wi-Fi network has an unlimited capacity, while this work characterizes the impact of the network capacity on the game's equilibrium. Third, Yu et al. did not consider the advertising's wear-out effect, and this work shows its impact on the optimal ad pricing. Riggins (2002) and Guo et al. (2017) also studied the comparison between the fee-based and ad-based services, but did not consider the competition between the service providers.

3. Model

In this section, we introduce the strategies of the decision makers (i.e., venue owners, advertiser, and users), and formulate their interactions as a three-stage game.

3.1. Venue Owners

Each venue owner needs to decide the advertising density and the ad price. We use $d_i \in \left[0,1\right]$ to denote venue owner i's advertising density $\left(i=1,2\right)$, which is the fraction of the time that a user needs to spend watching ads when using venue owner i's network. Then, we use $p_i > 0$ to denote venue owner i's unit ad price. Moreover, we assume that the two venue owners' Wi-Fi networks have the same capacity, and we capture it by $B > 0$.

3.2. Advertiser

We consider a single advertiser who wants to display its ads using both venue owners' Wi-Fi networks. Intuitively, the insights derived in our analysis can be easily extended to the case where there are multiple homogeneous advertisers. One interesting direction to extend our work is to consider multiple advertisers who have heterogeneous valuations for displaying ads.

We use $m_i \in [0,1]$ to denote the fraction of the time that the advertiser's ads are displayed to each user using venue owner i's Wi-Fi network. We define the advertiser's payoff as follows:

$$\Pi^{ad} \triangleq \sum_{i=1,2} x_i \left(f(m_i) - m_i p_i \right).$$

Here, $x_i \in [0,1]$ is venue owner i's market share, i.e., the fraction of users using its Wi-Fi network. The value of x_i is determined by the equilibrium of all users' choices, and will be given in the later analysis. We use function $f(\cdot)$ to represent the advertising effectiveness, which captures the improvement in a user's willingness to buy the advertiser's products. In this work, we consider a quadratic form for the advertising effectiveness: $f(m_i) = -Am_i^2 + Dm_i$, where $A \geq 0$. Here, parameter A captures the wear-out effect of advertising, i.e., as the number of ad repetition increases, the advertising may become annoying and a user's willingness to buy the products may decrease (see Pechmann and Stewart 1988). In this work, we consider the case where the advertiser has the same advertising effectiveness function for both venues' users. As an extension, we can consider heterogeneous advertising effectiveness functions. The second term in the advertiser's payoff is its payment to the venue owner i.

3.3. Users

We consider a continuum of users, i.e., each user has a negligible impact on the equilibrium. We normalize the mass of the users to 1. Each user has a reservation price for using the Wi-Fi network, and we assume the users' reservation prices are uniformly distributed in $[0,1]$. Each user decides whether to use Wi-Fi and which venue's network to use by comparing its reservation price and the *delivered prices* of the two Wi-Fi networks. Specifically, a Wi-Fi network's delivered price is the sum of the network's advertising density and congestion level. If and only if at least one network's delivered price is no greater than a user's reservation price, the user will use the Wi-Fi network. Moreover, the user will choose the network with a smaller delivered price. Recall that x_i is venue owner i's market share, B is the network capacity, and d_i is venue owner i's advertising density. We have the following Wardrop equilibrium, which describes the users' network selection choices:

$$\begin{cases} \dfrac{x_i}{B}+d_i \geq 1-x_1-x_2, & i=1,2, \\ x_i\left(\dfrac{x_i}{B}+d_i-1+x_1+x_2\right)=0, & i=1,2. \end{cases}$$

We first explain the inequality $\frac{x_i}{B}+d_i \geq 1-x_1-x_2$. At the equilibrium, the mass of users that do not select any of the two networks is $1-x_1-x_2$, and the highest reservation price of these users is also $1-x_1-x_2$. Therefore, the delivered prices (i.e., $\frac{x_i}{B}+d_i$) of the two networks at the equilibrium should be no smaller than this value. Second, we explain $x_i\left(\frac{x_i}{B}+d_i-1+x_1+x_2\right)=0$. This implies that when the venue owner i 's market share is positive, its delivered price should be $1-x_1-x_2$; when $\frac{x_i}{B}+d_i > 1-x_1-x_2$, the venue owner i 's market share is zero.

3.4. Three-Stage Game

We formulate the interactions among the decision markers by a three-stage game. In Stage I, the venue owners decide the advertising density d_i. In Stage II, they decide the unit ad price p_i. In Stage III, the advertiser decides m_i and the users make the network selection decisions.

4. Equilibrium Analysis

In this section, we analyze the three-stage game's equilibrium by backward induction.

4.1. Stage III: Users' Decisions

We characterize the users' Wardrop equilibrium in the following proposition.

Proposition 1. Given d_1 and d_2, there are three cases for the users' equilibrium:

(i) If $d_1 < \frac{1+Bd_2}{B+1}$ and $d_2 < \frac{1+Bd_1}{B+1}$, then $x_1^*(d_1,d_2)=\frac{B\left(1+Bd_2-(B+1)d_1\right)}{2B+1} > 0$ and

$$x_2^*(d_1,d_2)=\frac{B\left(1+Bd_1-(B+1)d_2\right)}{2B+1}>0;$$

(ii) If $d_2 \geq \frac{1+Bd_1}{B+1}$, then $x_1^*(d_1,d_2)=\frac{B(1-d_1)}{B+1}>0$ and $x_2^*(d_1,d_2)=0$;

(iii) If $d_1 \geq \frac{1+Bd_2}{B+1}$, then $x_2^*(d_1,d_2)=\frac{B(1-d_2)}{B+1}>0$ and $x_1^*(d_1,d_2)=0$.

Proposition 1 implies that when the two venue owners' advertising densities are close to each other, both of their market shares are positive. When a venue owner's advertising density is much smaller, the other venue owner's market share becomes zero. Note that both $\frac{1+Bd_2}{B+1}$ and $\frac{1+Bd_1}{B+1}$ decrease with B. Therefore, as the network capacity increases, the users become more sensitive to the difference between the advertising densities. When the capacity is large, even a slightly larger advertising density may lead to a zero market share for a venue owner.

4.2. Stage III: Advertiser's Decisions

We compute the advertiser's optimal choice of m_i in the following proposition.

Proposition 2. Given p_1 and p_2, we have $m_i^*(p_i)=\frac{\max\{D-p_i,0\}}{2A}$, for $i=1,2$.

Hence, the value of m_i^* decreases with the unit ad price and the wear-out effect's degree.

4.3. Stage II: Venue Owners' Ad Pricing

Venue owner i's revenue is the overall payment from the advertiser. Its problem in Stage II is formulated as follows:

$$\max_{p_i>0} x_i^*(d_1,d_2)m_i^*(p_i)p_i \qquad \text{s.t.} \quad m_i^*(p_i)\leq d_i.$$

In Stage II, the venue owners' decisions of d_1 and d_2 in Stage I are given. The constraint $m_i^*(p_i)\leq d_i$ means that the venue owner cannot sell more ads than it can display. We characterize the venue owner's optimal ad price as follows.

Proposition 3. Given d_i, venue owner i's optimal ad price is $p_i^*\left(d_i\right)=\max\left\{D-2Ad_i,\frac{D}{2}\right\}$.

We can see that when the wear-out effect's degree is small, the optimal ad price decreases with the wear-out effect's degree and also the advertising density d_i. As the wear-out effect becomes large, the venue owner needs to decrease its price to incentivize the advertiser to display ads. As the advertising density increases, the venue owner can display more ads and hence will decrease its ad price. When the wear-out effect's degree is large, the optimal ad price is independent of the wear-out effect's degree and the advertising density.

4.4. Stage I: Venue Owners' Advertising Densities

In Stage I, venue owner i's revenue is computed as $\Pi_i\left(d_1,d_2\right)=x_i^*\left(d_1,d_2\right)m_i^*\left(p_i^*\left(d_i\right)\right)p_i^*\left(d_i\right)$. Note that $x_i^*\left(d_1,d_2\right)$ is in Proposition 1, $m_i^*\left(p_i^*\left(d_i\right)\right)$ is in Proposition 2, and $p_i^*\left(d_i\right)$ is in Proposition 3. Each venue owner decides its advertising density to respond to the other venue owner's advertising density. We will analyze d_1^* and d_2^* via numerical experiments, and study their dependence on the parameters, such as B and A.

5. Conclusion

In this work, we studied the duopoly competition in advertising-sponsored Wi-Fi provision, and characterized the impacts of the network capacity and advertising's wear-out effect on the game equilibrium. We plan to further analyze the venue owners' optimal choices of advertising densities and consider the heterogeneous network capacity situation.

6. References

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